Useful Inequalities $\{x^2 \geqslant 0\}$ v0.44 · January 27, 2026

Cauchy-Schwarz $\left(\sum_{i=1}^n x_i y_i\right)^2 \le \left(\sum_{i=1}^n x_i^2\right)\left(\sum_{i=1}^n y_i^2\right)$

Minkowski $\left(\sum_{i=1}^n |x_i + y_i|^p\right)^{\frac{1}{p}} \le \left(\sum_{i=1}^n |x_i|^p\right)^{\frac{1}{p}} + \left(\sum_{i=1}^n |y_i|^p\right)^{\frac{1}{p}}$ for $p \ge 1$.

Hölder $\sum_{i=1}^n |x_i y_i| \le \left(\sum_{i=1}^n |x_i|^p\right)^{1/p}\left(\sum_{i=1}^n |y_i|^q\right)^{1/q}$ for $p, q > 1,\ \frac{1}{p} + \frac{1}{q} = 1$.

Bernoulli $(1+x)^r \ge 1 + rx$ for $x \ge -1,\ r \in \mathbb{R} \setminus (0,1)$. Reverse for $r \in [0,1]$.

$(1+x)^r \le \frac{1}{1-rx}$ for $x \in [-1, \frac{1}{r}),\ r \ge 0$.

$(1+x)^r \le 1 + \frac{x}{x+1} r$ for $x \ge 0,\ r \in [-1, 0]$.

$(1+x)^r \le 1 + (2^r - 1)x$ for $x \in [0,1],\ r \in \mathbb{R} \setminus (0,1)$.

$(1+nx)^{n+1} \ge (1 + (n+1)x)^n$ for $x \ge 0,\ n \in \mathbb{N}$.

$(a+b)^n \le a^n + nb(a+b)^{n-1}$ for $a, b \ge 0,\ n \in \mathbb{N}$.

exponential $e^x \ge \left(1 + \frac{x}{n}\right)^n \ge 1 + x$; $\left(1 + \frac{x}{n}\right)^n \ge e^x\left(1 - \frac{x^2}{n}\right)$ for $n \ge 1,\ |x| \le n$.

$\frac{x^n}{n!} + 1 \le e^x \le \left(1 + \frac{x}{n}\right)^{n + x/2}$; $e^x \ge \left(\frac{ex}{n}\right)^n$ for $x, n > 0$.

$x^y + y^x > 1$; $x^y > \frac{x}{x+y}$; $e^x > \left(1 + \frac{x}{y}\right)^y > e^{\frac{xy}{x+y}}$; $\frac{x}{y} \ge e^{\frac{x-y}{x}}$ for $x, y > 0$.

$\frac{1}{2-x} < x^x < x^2 - x + 1$; $e^{2x} \le \frac{1+x}{1-x}$ for $x \in (0,1)$.

$x^{1/r}(x-1) \le rx(x^{1/r} - 1)$ for $x, r \ge 1$; $2^{-x} \le 1 - \frac{x}{2}$ for $x \in [0,1]$.

$xe^x \ge x + x^2 + \frac{x^3}{2}$; $e^x \le x + e^{x^2}$; $e^x + e^{-x} \le 2e^{x^2/2}$ for $x \in \mathbb{R}$.

$e^{-x} \le 1 - \frac{x}{2}$ for $x \in [0, 1.59]$; $e^x \le 1 + x + x^2$ for $x < 1.79$.

$\left(1 + \frac{x}{p}\right)^p \ge \left(1 + \frac{x}{q}\right)^q$ for $(i)\ x > 0,\ p > q > 0$,

$(ii)\ -p < -q < x < 0$, $(iii)\ -q > -p > x > 0$. Reverse for:

$(iv)\ q < 0 < p$, $-q > x > 0$, $(v)\ q < 0 < p$, $-p < x < 0$.

logarithm $\frac{x}{1+x} \le \ln(1+x) \le \frac{x(6+x)}{6+4x} \le x$ for $x > -1$.

$\frac{2}{2+x} \le \frac{1}{\sqrt{1 + x + x^2/12}} \le \frac{\ln(1+x)}{x} \le \frac{1}{\sqrt{x+1}} \le \frac{2+x}{2+2x}$ for $x > -1$.

$\ln(n) + \frac{1}{n+1} < \ln(n+1) < \ln(n) + \frac{1}{n} \le \sum_{i=1}^n \frac{1}{i} \le \ln(n) + 1$ for $n \ge 1$.

$|\ln x| \le \frac{1}{2}|x - \frac{1}{x}|$; $\ln(x+y) \le \ln(x) + \frac{y}{x}$; $\ln x \le y(x^{\frac{1}{y}} - 1)$; $x, y > 0$.

$\ln(1+x) \ge x - \frac{x^2}{2}$ for $x \ge 0$; $\ln(1+x) \ge x - x^2$ for $x \ge -0.68$.

trigonometric $x - \frac{x^3}{2} \le x\cos x \le \frac{x\cos x}{1 - x^2/3} \le x\sqrt[3]{\cos x} \le x - x^3/6 \le x\cos\frac{x}{\sqrt{3}} \le \sin x$,

hyperbolic $x\cos x \le \frac{x^3}{\sinh^2 x} \le x\cos^2(x/2) \le \sin x \le (x\cos x + 2x)/3 \le \frac{x^2}{\sinh x}$,

$\max\left\{\frac{2}{\pi}, \frac{\pi^2 - x^2}{\pi^2 + x^2}\right\} \le \frac{\sin x}{x} \le \cos\frac{x}{2} \le 1 \le 1 + \frac{x^2}{3} \le \frac{\tan x}{x}$ for $x \in [0, \frac{\pi}{2}]$.

square root $2\sqrt{x+1} - 2\sqrt{x} < \frac{1}{\sqrt{x}} < \sqrt{x+1} - \sqrt{x-1} < 2\sqrt{x} - 2\sqrt{x-1}$ for $x \ge 1$.

$x \le \frac{x+1}{2} - \frac{(x-1)^2}{2} \le \sqrt{x} \le \frac{x+1}{2} - \frac{(x-1)^2}{8}$ for $x \in [0,1]$.

binomial $\max\{\frac{n^k}{k^k}, \frac{(n-k+1)^k}{k!}\} \le \binom{n}{k} \le \frac{n^k}{k!} \le \left(\frac{en}{k}\right)^k$; $\binom{n}{k} \le \frac{n^n}{k^k (n-k)^{n-k}}$.

$\frac{n^k}{4k!} \le \binom{n}{k}$ for $\sqrt{n} \ge k \ge 0$; $\frac{4^n}{\sqrt{\pi n}}(1 - \frac{1}{8n}) \le \binom{2n}{n} \le \frac{4^n}{\sqrt{\pi n}}(1 - \frac{1}{9n})$.

$\binom{n_1}{k_1}\binom{n_2}{k_2} \le \binom{n_1+n_2}{k_1+k_2}$; $\binom{tn}{k} \ge t^k \binom{n}{k}$ for $t \ge 1$.

$\frac{\sqrt{\pi}}{2} G \le \binom{n}{\alpha n} \le G$ for $G = \frac{2^{nH(\alpha)}}{\sqrt{2\pi n \alpha(1-\alpha)}}$, $H(x) = -\lg(x^x(1-x)^{1-x})$.

$\sum_{i=0}^d \binom{n}{i} \le \min\left\{n^d + 1,\ \left(\frac{en}{d}\right)^d,\ 2^n\right\}$ for $n \ge d \ge 1$.

$\sum_{i=0}^{\alpha n} \binom{n}{i} \le \min\left\{\frac{1-\alpha}{1-2\alpha}\binom{n}{\alpha n},\ 2^{nH(\alpha)},\ 2^n e^{-2n\left(\frac{1}{2} - \alpha\right)^2}\right\}$ for $\alpha \in (0, \frac{1}{2})$.

binary entropy $H(x) \le \min\{x \lg \frac{4}{x}, (1-x)\lg\frac{4}{1-x}, (4x(1-x))^{1/\ln 4}, \lg x \cdot \lg(1-x)\}$,

$H(x) \ge \max\{4x(1-x),\ \ln 2 \cdot \lg x \cdot \lg(1-x)\}$ for $x \in (0,1)$.

$1 - 5x^2 \le H(1/2 - x) \le 1 - x^2$ for $0 < x \le 1/4$.

Stirling $e\left(\frac{n}{e}\right)^n \le \sqrt{2\pi n}\left(\frac{n}{e}\right)^n e^{1/(12n+1)} \le n! \le \sqrt{2\pi n}\left(\frac{n}{e}\right)^n e^{1/12n} \le en\left(\frac{n}{e}\right)^n$

means $\min x_i \le \frac{n}{\sum x_i^{-1}} \le (\prod x_i)^{1/n} \le \frac{1}{n}\sum x_i \le \sqrt{\frac{1}{n}\sum {x_i}^2} \le \frac{\sum x_i^2}{\sum x_i} \le \max x_i$

$\min_i \frac{|x_i|}{|y_i|} \le \left(\sum_i |x_i|\right) / \left(\sum_i |y_i|\right) \le \max_i \frac{|x_i|}{|y_i|}$

power means $M_p \le M_q$ for $p \le q$, where $M_p = \left(\sum_i w_i |x_i|^p\right)^{1/p}$, $w_i \ge 0$, $\sum_i w_i = 1$.
In the limit $M_0 = \prod_i |x_i|^{w_i}$, $M_{-\infty} = \min_i\{x_i\}$, $M_\infty = \max_i\{x_i\}$.

Lehmer $\frac{\sum_i w_i |x_i|^p}{\sum_i w_i |x_i|^{p-1}} \le \frac{\sum_i w_i |x_i|^q}{\sum_i w_i |x_i|^{q-1}}$ for $p \le q$, $w_i \ge 0$.

log mean $\sqrt{xy} \le \left(\frac{\sqrt{x} + \sqrt{y}}{2}\right)(xy)^{\frac{1}{4}} \le \frac{x-y}{\ln(x) - \ln(y)} \le \left(\frac{\sqrt{x} + \sqrt{y}}{2}\right)^2 \le \frac{x+y}{2}$ for $x, y > 0$.

Heinz $\sqrt{xy} \le \frac{x^{1-\alpha}y^\alpha + x^\alpha y^{1-\alpha}}{2} \le \frac{x+y}{2}$ for $x, y > 0$, $\alpha \in [0,1]$.

Maclaurin-Newton ${S_k}^2 \ge S_{k-1} S_{k+1}$ and $(S_k)^{1/k} \ge (S_{k+1})^{1/(k+1)}$ for $1 \le k < n$,
$S_k = \frac{1}{\binom{n}{k}} \sum_{1 \le i_1 < \cdots < i_k \le n} a_{i_1} a_{i_2} \cdots a_{i_k}$, and $a_i \ge 0$.

Jensen $\varphi\left(\sum_i p_i x_i\right) \le \sum_i p_i \varphi(x_i)$ where $p_i \ge 0$, $\sum p_i = 1$, and φ convex.
Alternatively: $\varphi(\mathrm{E}[X]) \le \mathrm{E}[\varphi(X)]$. For concave φ the reverse holds.

Chebyshev $\sum_{i=1}^n f(x_i) g(x_i) p_i \ge \left(\sum_{i=1}^n f(x_i) p_i\right)\left(\sum_{i=1}^n g(x_i) p_i\right)$
for $x_1 \le \cdots \le x_n$ and f, g nondecreasing, $p_i \ge 0$, $\sum p_i = 1$.
Alternatively: $\mathrm{E}[f(X)g(X)] \ge \mathrm{E}[f(X)]\mathrm{E}[g(X)]$.

rearrangement $\sum_{i=1}^n a_i b_i \ge \sum_{i=1}^n a_i b_{\pi(i)} \ge \sum_{i=1}^n a_i b_{n-i+1}$ for $a_1 \le \cdots \le a_n$,
$b_1 \le \cdots \le b_n$ and π a permutation of $[n]$. More generally:
$\sum_{i=1}^n f_i(b_i) \ge \sum_{i=1}^n f_i(b_{\pi(i)}) \ge \sum_{i=1}^n f_i(b_{n-i+1})$
with $\left(f_{i+1}(x) - f_i(x)\right)$ nondecreasing for all $1 \le i < n$.
Dually: $\prod_{i=1}^n (a_i + b_i) \le \prod_{i=1}^n (a_i + b_{\pi(i)}) \le \prod_{i=1}^n (a_i + b_{n-i+1})$ for $a_i, b_i \ge 0$.

Weierstrass

$1-\sum_i w_i x_i \le \prod_i (1-x_i)^{w_i}$, and
$1+\sum_i w_i x_i \le \prod_i (1+x_i)^{w_i} \le \prod_i (1-x_i)^{-w_i}$ for $x_i \in [0,1]$, $w_i \ge 1$.
$\prod_i (1 \pm x_i)^{w_i} \le (1 \mp \sum_i w_i x_i)^{-1}$ if additionally $\sum_i w_i x_i \le 1$.

Kantorovich

$(\sum_i x_i^2)(\sum_i y_i^2) \le \left(\frac{A}{G}\right)^2 (\sum_i x_i y_i)^2$ for $x_i, y_i > 0$,
$0 < m \le \frac{x_i}{y_i} \le M < \infty$, $A = (m+M)/2$, $G = \sqrt{mM}$.

Nesbitt

$\sum_{i=1}^n \frac{a_i}{S-a_i} \ge \frac{n}{n-1}$ and $\sum_{i=1}^n \frac{S}{S-a_i} \ge \frac{n^2}{n-1}$ for $a_i \ge 0$, $S = \sum a_i$.

sum & integral

$\int_{L-1}^U f(x)\,dx \le \sum_{i=L}^U f(i) \le \int_L^{U+1} f(x)\,dx$ for f nondecreasing.

Cauchy

$f'(a) \le \frac{f(b)-f(a)}{b-a} \le f'(b)$ where $a < b$, and f convex.

Hermite

$\varphi\left(\frac{a+b}{2}\right) \le \frac{1}{b-a}\int_a^b \varphi(x)\,dx \le \frac{\varphi(a)+\varphi(b)}{2}$ for φ convex.

Gibbs

$\sum_i a_i \log \frac{a_i}{b_i} \ge a \log \frac{a}{b}$ for $a_i, b_i \ge 0$, or more generally:
$\sum_i a_i \varphi\left(\frac{b_i}{a_i}\right) \le a\,\varphi\left(\frac{b}{a}\right)$ for φ concave, and $a = \sum a_i$, $b = \sum b_i$.

Pinsker

$\sum_i a_i \lg \frac{a_i}{b_i} \ge \frac{1}{2}\left(\sum_i |a_i - b_i|\right)^2$ for $a_i, b_i \ge 0$, $\sum a_i = \sum b_i = 1$.

Chong

$\sum_{i=1}^n \frac{a_i}{a_{\pi(i)}} \ge n$ and $\prod_{i=1}^n a_i^{a_i} \ge \prod_{i=1}^n a_i^{a_{\pi(i)}}$ for $a_i > 0$.

Schur-Vornicu

$f(x)(x-y)^k(x-z)^k + f(y)(y-z)^k(y-x)^k + f(z)(z-x)^k(z-y)^k \ge 0$
where $x, y, z \ge 0$, $k \ge 1$ integer, f convex or monotonic, $f \ge 0$.

Young

$(\frac{1}{px^p} + \frac{1}{qy^q})^{-1} \le xy \le \frac{x^p}{p} + \frac{y^q}{q}$ for $x, y, p, q > 0$, $\frac{1}{p} + \frac{1}{q} = 1$.
$\int_0^a f(x)\,dx + \int_0^b f^{-1}(x)\,dx \ge ab$, for f cont., strictly increasing.

Shapiro

$\sum_{i=1}^n \frac{x_i}{x_{i+1}+x_{i+2}} \ge \frac{n}{2}$ where $x_i > 0$, $(x_{n+1}, x_{n+2}) := (x_1, x_2)$,
and $n \le 12$ if even, $n \le 23$ if odd.

Hadamard

$(\det A)^2 \le \prod_{i=1}^n \sum_{j=1}^n A_{ij}^2$ where A is an $n \times n$ matrix.

Schur

$\sum_{i=1}^n \lambda_i^2 \le \sum_{i=1}^n \sum_{j=1}^n A_{ij}^2$ and $\sum_{i=1}^k d_i \le \sum_{i=1}^k \lambda_i$ for $1 \le k \le n$.
A is an $n \times n$ matrix. For the second inequality A is symmetric.
$\lambda_1 \ge \cdots \ge \lambda_n$ the eigenvalues, $d_1 \ge \cdots \ge d_n$ the diagonal elements.

Ky Fan

$\frac{\prod_{i=1}^n x_i^{a_i}}{\prod_{i=1}^n (1-x_i)^{a_i}} \le \frac{\sum_{i=1}^n a_i x_i}{\sum_{i=1}^n a_i (1-x_i)}$ for $x_i \in [0, \frac{1}{2}]$, $a_i \in [0,1]$, $\sum a_i = 1$.

Aczél

$(a_1 b_1 - \sum_{i=2}^n a_i b_i)^2 \ge (a_1^2 - \sum_{i=2}^n a_i^2)(b_1^2 - \sum_{i=2}^n b_i^2)$
given that $a_1^2 > \sum_{i=2}^n a_i^2$ or $b_1^2 > \sum_{i=2}^n b_i^2$.

Mahler

$\prod_{i=1}^n (x_i + y_i)^{1/n} \ge \prod_{i=1}^n x_i^{1/n} + \prod_{i=1}^n y_i^{1/n}$ where $x_i, y_i > 0$.

Abel

$b_1 \cdot \min_k \sum_{i=1}^k a_i \le \sum_{i=1}^n a_i b_i \le b_1 \cdot \max_k \sum_{i=1}^k a_i$ for $b_1 \ge \cdots \ge b_n \ge 0$.

Milne

$\left(\sum_{i=1}^n (a_i + b_i)\right)\left(\sum_{i=1}^n \frac{a_i b_i}{a_i + b_i}\right) \le \left(\sum_{i=1}^n a_i\right)\left(\sum_{i=1}^n b_i\right)$ for $a_i, b_i \ge 0$.

Carleman

$\sum_{k=1}^n \left(\prod_{i=1}^k |a_i|\right)^{1/k} \le e \sum_{k=1}^n |a_k|$

sum & product

$\left|\prod_{i=1}^n a_i - \prod_{i=1}^n b_i\right| \le \sum_{i=1}^n |a_i - b_i|$ for $|a_i|, |b_i| \le 1$.
$\prod_{i=1}^n (t + a_i) \ge (t+1)^n$ where $\prod_{i=1}^n a_i \ge 1$, $a_i > 0$, $t > 0$.

Radon

$\sum_i \frac{x_i^p}{a_i^{p-1}} \ge \frac{(\sum_i x_i)^p}{(\sum_i a_i)^{p-1}}$ for $x_i, a_i \ge 0$, $p \ge 1$ (rev. if $p \in [0,1]$).

Karamata

$\sum_{i=1}^n \varphi(a_i) \ge \sum_{i=1}^n \varphi(b_i)$ for $a_1 \ge a_2 \ge \cdots \ge a_n$, $b_1 \ge \cdots \ge b_n$,
and $\{a_i\} \succeq \{b_i\}$ (majorization), i.e. $\sum_{i=1}^t a_i \ge \sum_{i=1}^t b_i$ for all $1 \le t \le n$,
with $\sum_{i=1}^n a_i = \sum_{i=1}^n b_i$, and φ convex (for concave φ the reverse holds).

Muirhead

$\sum_\pi x_{\pi(1)}^{a_1} \cdots x_{\pi(n)}^{a_n} \ge \sum_\pi x_{\pi(1)}^{b_1} \cdots x_{\pi(n)}^{b_n}$, sums over permut. π of $[n]$,
where $a_1 \ge \cdots \ge a_n$, $b_1 \ge \cdots \ge b_n$, $\{a_k\} \succeq \{b_k\}$, $x_i \ge 0$.

Hilbert

$\sum_{m=1}^\infty \sum_{n=1}^\infty \frac{a_m b_n}{m+n} \le \pi \left(\sum_{m=1}^\infty a_m^2\right)^{\frac{1}{2}} \left(\sum_{n=1}^\infty b_n^2\right)^{\frac{1}{2}}$ for $a_m, b_n \in \mathbb{R}$.
With $\max\{m,n\}$ instead of $m+n$, we have 4 instead of π.

Hardy

$\sum_{n=1}^\infty \left(\frac{a_1 + a_2 + \cdots + a_n}{n}\right)^p \le \left(\frac{p}{p-1}\right)^p \sum_{n=1}^\infty a_n^p$ for $a_n \ge 0$, $p > 1$.

Mathieu

$\frac{1}{c^2 + 1/2} < \sum_{n=1}^\infty \frac{2n}{(n^2+c^2)^2} < \frac{1}{c^2}$ for $c \ne 0$.

Kraft

$\sum 2^{-c(i)} \le 1$ for $c(i)$ depth of leaf i of binary tree, sum over all leaves.

LYM

$\sum_{X \in \mathcal{A}} \binom{n}{|X|}^{-1} \le 1$, $\mathcal{A} \subset 2^{[n]}$, no set in $\mathcal{A}$ is subset of another set in $\mathcal{A}$.

FKG

$2^n \cdot |\mathcal{A} \cap \mathcal{B}| \ge |\mathcal{A}| \cdot |\mathcal{B}|$, for $\mathcal{A}, \mathcal{B}$ monotone set systems over $\{1, \ldots, n\}$.

Shearer

$|\mathcal{A}|^t \le \prod_{F \in \mathcal{F}} |\mathrm{trace}_F(\mathcal{A})|$ for $\mathcal{A}, \mathcal{F} \subseteq 2^{[n]}$, where every $i \in [n]$
appears in at least t sets of $\mathcal{F}$, and $\mathrm{trace}_F(\mathcal{A}) = \{F \cap A : A \in \mathcal{A}\}$.

Sauer-Shelah

$|\mathcal{A}| \le |\mathrm{str}(\mathcal{A})| \le \sum_{i=0}^{\mathrm{vc}(\mathcal{A})} \binom{n}{i} \le n^{\mathrm{vc}(\mathcal{A})} + 1$ for $\mathcal{A} \subseteq 2^{[n]}$, and
$\mathrm{str}(\mathcal{A}) = \{X \subseteq [n] : \mathrm{trace}_X(\mathcal{A}) = 2^X\}$, $\mathrm{vc}(\mathcal{A}) = \max\{|X| : X \in \mathrm{str}(\mathcal{A})\}$.

Khintchine

$\sqrt{\sum_i a_i^2} \ge \mathrm{E}\left[\left|\sum_i a_i r_i\right|\right] \ge \frac{1}{\sqrt{2}}\sqrt{\sum_i a_i^2}$ where $a_i \in \mathbb{R}$, and
$r_i \in \{\pm 1\}$ random variables (r.v.) i.i.d. w.pr. $\frac{1}{2}$.

Bonferroni

$\Pr\left[\bigvee_{i=1}^n A_i\right] \le \sum_{j=1}^k (-1)^{j-1} S_j$ for $1 \le k \le n$, k odd (rev. for k even),
$S_k = \sum_{1 \le i_1 < \cdots < i_k \le n} \Pr\left[A_{i_1} \wedge \cdots \wedge A_{i_k}\right]$ where A_i are events.

Bhatia-Davis

$\mathrm{Var}[X] \le (M - \mathrm{E}[X])(\mathrm{E}[X] - m)$ where $X \in [m, M]$.

Samuelson

$$\mu - \sigma\sqrt{n-1} \le x_i \le \mu + \sigma\sqrt{n-1} \quad \text{for } i = 1, \dots, n,$$
where $\mu = \sum x_i/n$, $\sigma^2 = \sum (x_i - \mu)^2/n$.

Markov

$$\Pr\big[|X| \ge a\big] \le \mathrm{E}\big[|X|\big]/a \quad \text{where } X \text{ is a r.v., } a > 0.$$
$$\Pr\big[X \le c\big] \le (1 - \mathrm{E}[X])/(1-c) \quad \text{for } X \in [0,1] \text{ and } c \in \big[0, \mathrm{E}[X]\big].$$
$$\Pr\big[X \in S\big] \le \mathrm{E}[f(X)]/s \quad \text{for } f \ge 0, \text{ and } f(x) \ge s > 0 \text{ for all } x \in S.$$

Chebyshev

$$\Pr\big[\big|X - \mathrm{E}[X]\big| \ge t\big] \le \mathrm{Var}[X]/t^2 \quad \text{where } t > 0.$$
$$\Pr\big[X - \mathrm{E}[X] \ge t\big] \le \mathrm{Var}[X]/(\mathrm{Var}[X] + t^2) \quad \text{where } t > 0.$$

2^{nd} *moment*

$$\Pr\big[X > 0\big] \ge (\mathrm{E}[X])^2/(\mathrm{E}[X^2]) \quad \text{where } \mathrm{E}[X] \ge 0.$$
$$\Pr\big[X = 0\big] \le \mathrm{Var}[X]/(\mathrm{E}[X^2]) \quad \text{where } \mathrm{E}[X^2] \ne 0.$$

k^{th} *moment*

$$\Pr\big[\big|X - \mu\big| \ge t\big] \le \frac{\mathrm{E}\left[(X-\mu)^k\right]}{t^k} \quad \text{and}$$
$$\Pr\big[\big|X - \mu\big| \ge t\big] \le C_k \left(\frac{nk}{et^2}\right)^{k/2} \quad \text{for } X_i \in [0,1] \text{ } k\text{-wise indep. r.v.,}$$
$X = \sum X_i$, $i = 1, \dots, n$, $\mu = \mathrm{E}[X]$, $C_k = 2\sqrt{\pi k} e^{1/6k}$, k even.

4^{th} *moment*

$$\mathrm{E}\big[|X|\big] \ge \frac{\left(\mathrm{E}\left[X^2\right]\right)^{3/2}}{\left(\mathrm{E}\left[X^4\right]\right)^{1/2}} \quad \text{where } 0 < \mathrm{E}\left[X^4\right] < \infty.$$

Chernoff

$$\Pr[X \ge t] \le F(a)/a^t \quad \text{for } X \text{ r.v., } \Pr[X = k] = p_k,$$
$F(z) = \sum_k p_k z^k$ probability gen. func., and $a \ge 1$.
$$\Pr\big[X \ge \Delta\big] = \Pr\big[X \le -\Delta\big] \le \exp\left(-\Delta^2/2n\right) \quad \text{for } X_i \in \{0,1\} \text{ i.r.v.,}$$
and $\Pr[X_i = \pm 1] = \frac{1}{2}$ for $i = 1, \dots, n$, $X = \sum X_i$, $\Delta \ge 0$.
$$\Pr\big[X \ge (1+\delta)\mu\big] \le \left(\frac{e^\delta}{(1+\delta)^{(1+\delta)}}\right)^\mu \le \exp\left(\frac{-\mu\delta^2}{2+\delta}\right)$$
$$\Pr\big[X \le (1-\delta)\mu\big] \le \left(\frac{e^{-\delta}}{(1-\delta)^{(1-\delta)}}\right)^\mu \le \exp\left(\frac{-\mu\delta^2}{2}\right)$$
for $X_i \in [0,1]$ i.r.v., $X = \sum X_i$, $\mu = \mathrm{E}[X]$, $\delta \ge 0$.
Further from the mean: $\Pr\big[X \ge R\big] \le 2^{-R}$ for $R \ge 2e\mu$ ($\approx 5.44\mu$).
$$\Pr\big[\textstyle\sum X_i \ge t\big] \le \binom{n}{k} p^k / \binom{t}{k} \quad \text{for } X_i \in \{0,1\} \text{ } k\text{-wise i.r.v., } \mathrm{E}[X_i] = p.$$
$$\Pr\big[X \ge (1+\delta)\mu\big] \le \binom{n}{\hat{k}} p^{\hat{k}} / \binom{(1+\delta)\mu}{\hat{k}} \quad \text{for } X_i \in [0,1] \text{ } k\text{-wise i.r.v.,}$$
$k \ge \hat{k} = \lceil \mu\delta/(1-p) \rceil$, $\mathrm{E}[X_i] = p_i$, $X = \sum X_i$, $\mu = \mathrm{E}[X]$, $p = \frac{\mu}{n}$, $\delta > 0$.

Hoeffding

$$\Pr\big[\big|X - \mathrm{E}[X]\big| \ge \delta\big] \le 2\exp\left(\frac{-2\delta^2}{\sum_{i=1}^n (b_i - a_i)^2}\right) \quad \text{for } X_i \text{ i.r.v.,}$$
$X_i \in [a_i, b_i]$ (w. prob. 1), $X = \sum X_i$, $\delta \ge 0$.
A related lemma, assuming $\mathrm{E}[X] = 0$, $X \in [a,b]$ (w. prob. 1) and $\lambda \in \mathbb{R}$:
$$\mathrm{E}\big[e^{\lambda X}\big] \le \exp\left(\frac{\lambda^2 (b-a)^2}{8}\right)$$

Kolmogorov

$$\Pr\big[\max_k |S_k| \ge \varepsilon\big] \le \tfrac{1}{\varepsilon^2}\mathrm{Var}[S_n] = \tfrac{1}{\varepsilon^2}\sum_i \mathrm{Var}[X_i]$$
where $X_1, \dots, X_n$ are i.r.v., $\mathrm{E}[X_i] = 0$,
$\mathrm{Var}[X_i] < \infty$ for all i, $S_k = \sum_{i=1}^k X_i$ and $\varepsilon > 0$.

Paley-Zygmund

$$\Pr\big[X \ge \delta \cdot \mathrm{E}[X]\big] \ge (1-\delta)^2 \frac{\mathrm{E}[X]^2}{\mathrm{E}[X^2]} \text{ for } X \ge 0,\ \mathrm{Var}[X] < \infty,\ \delta \in (0,1).$$

Vysochanskij-Petunin-Gauss

$$\Pr\big[\big|X - \mathrm{E}[X]\big| \ge \lambda\sigma\big] \le \tfrac{4}{9\lambda^2} \quad \text{if } \lambda \ge \sqrt{\tfrac{8}{3}},$$
$$\Pr\big[\big|X - m\big| \ge \varepsilon\big] \le \tfrac{4\tau^2}{9\varepsilon^2} \quad \text{if } \varepsilon \ge \tfrac{2\tau}{\sqrt{3}},$$
$$\Pr\big[\big|X - m\big| \ge \varepsilon\big] \le 1 - \tfrac{\varepsilon}{\sqrt{3}\tau} \quad \text{if } \varepsilon \le \tfrac{2\tau}{\sqrt{3}}.$$
Where X is a unimodal r.v. with mode m,
$\sigma^2 = \mathrm{Var}[X] < \infty$, $\tau^2 = \mathrm{Var}[X] + (\mathrm{E}[X] - m)^2 = \mathrm{E}[(X-m)^2]$.

Etemadi

$$\Pr\big[\max_{1 \le k \le n} |S_k| \ge 3\alpha\big] \le 3 \max_{1 \le k \le n} \big(\Pr\big[\,|S_k| \ge \alpha\big]\big)$$
where X_i are i.r.v., $S_k = \sum_{i=1}^k X_i$, $\alpha \ge 0$.

Doob

$$\Pr\big[\max_{1 \le k \le n} |X_k| \ge \varepsilon\big] \le \mathrm{E}\big[|X_n|\big]/\varepsilon \quad \text{for martingale } (X_k) \text{ and } \varepsilon > 0.$$

Bennett

$$\Pr\big[\textstyle\sum_{i=1}^n X_i \ge \varepsilon\,\big] \le \exp\left(-\frac{n\sigma^2}{M^2}\,\theta\big(\frac{M\varepsilon}{n\sigma^2}\big)\right) \quad \text{where } X_i \text{ i.r.v.,}$$
$\mathrm{E}[X_i] = 0$, $\sigma^2 = \frac{1}{n}\sum \mathrm{Var}[X_i]$, $|X_i| \le M$ (w. prob. 1), $\varepsilon \ge 0$,
$\theta(u) = (1+u)\log(1+u) - u$.

Bernstein

$$\Pr\big[\textstyle\sum_{i=1}^n X_i \ge \varepsilon\,\big] \le \exp\left(\frac{-\varepsilon^2}{2(n\sigma^2 + M\varepsilon/3)}\right) \quad \text{for } X_i \text{ i.r.v.,}$$
$\mathrm{E}[X_i] = 0$, $|X_i| < M$ (w. prob. 1) for all i, $\sigma^2 = \frac{1}{n}\sum \mathrm{Var}[X_i]$, $\varepsilon \ge 0$.

Azuma

$$\Pr\big[\big|X_n - X_0\big| \ge \delta\big] \le 2\exp\left(\frac{-\delta^2}{2\sum_{i=1}^n {c_i}^2}\right) \quad \text{for martingale } (X_k) \text{ s.t.}$$
$\big|X_i - X_{i-1}\big| < c_i$ (w. prob. 1), for $i = 1, \dots, n$, $\delta \ge 0$.

Efron-Stein

$$\mathrm{Var}[Z] \le \tfrac{1}{2}\,\mathrm{E}\left[\sum_{i=1}^n \left(Z - Z^{(i)}\right)^2\right] \quad \text{for } X_i, {X_i}' \in \mathcal{X} \text{ i.r.v.,}$$
$f : \mathcal{X}^n \to \mathbb{R}$, $Z = f(X_1, \dots, X_n)$, $Z^{(i)} = f(X_1, \dots, {X_i}', \dots, X_n)$.

McDiarmid

$$\Pr\big[\big|Z - \mathrm{E}[Z]\big| \ge \delta\big] \le 2\exp\left(\frac{-2\delta^2}{\sum_{i=1}^n c_i^2}\right) \quad \text{for } X_i, {X_i}' \in \mathcal{X} \text{ i.r.v.,}$$
Z, $Z^{(i)}$ as before, s.t. $\big|Z - Z^{(i)}\big| \le c_i$ for all i, and $\delta \ge 0$.

Janson

$$M \le \Pr\big[\textstyle\bigwedge \overline{B}_i\big] \le M\exp\left(\frac{\Delta}{2-2\varepsilon}\right) \quad \text{where } \Pr[B_i] \le \varepsilon \text{ for all } i,$$
$M = \prod(1 - \Pr[B_i])$, $\Delta = \sum_{i \ne j,\, B_i \sim B_j} \Pr[B_i \wedge B_j]$.

Lovász

$$\Pr\big[\textstyle\bigwedge \overline{B}_i\big] \ge \prod(1 - x_i) > 0 \quad \text{where } \Pr[B_i] \le x_i \cdot \prod_{(i,j) \in D} (1 - x_j),$$
for $x_i \in [0,1)$ for all $i = 1, \dots, n$ and D the dependency graph.
If each B_i mutually indep. of all other events, except at most d,
$\Pr[B_i] \le p$ for all $i = 1, \dots, n$, then if $ep(d+1) \le 1$ then $\Pr\big[\bigwedge \overline{B}_i\big] > 0$.

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