Self-adaptation via Multi-objectivisation: Multi-Objective Self-Adaptive Evolutionary Algorithms (MOSA-EA)

Xiaoyu Qin*

Theory of Evolutionary Computation Group
School of Computer Science
University of Birmingham
United Kingdom

22 March 2023

Adelaide, Australia

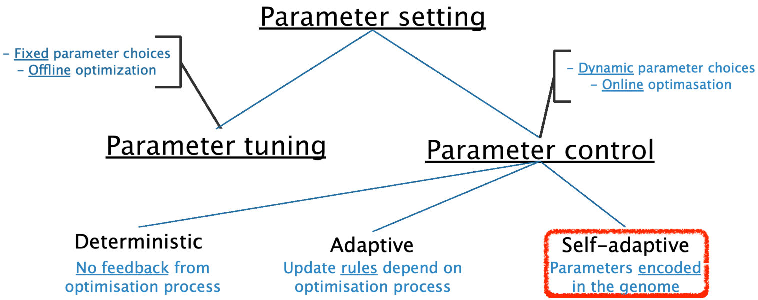
*xxq896@cs.bham.ac.uk

Content

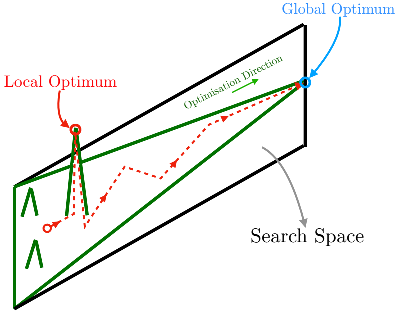
Background

Parameter Settings in EAs

- EAs are parameterised algorithms.
- Parameter setting can dramatically impact the performance of EAs (Doerr and Doerr, 2020). $\Longrightarrow$ **IMPORTANCE**
- Parameter setting is instance- and state-dependent (Doerr and Doerr, 2020). $\Longrightarrow$ **DIFFICULTY**
- Classification scheme of parameter setting (Eiben et al., 1999):

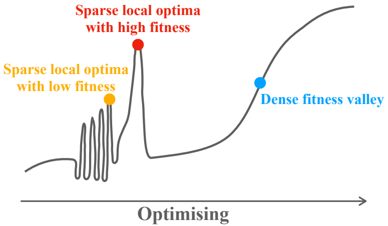


Sparse Local Optima



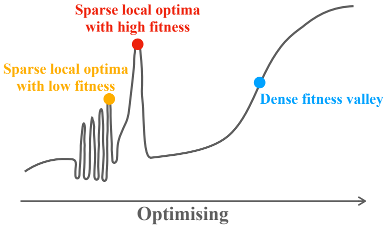
- Elitist EAs can get stuck on local optima (Jagerskupper and Storch, 2007; Dang et al., 2020; Doerr, 2022; Dang et al., 2021).
- *Sparse deceptive regions* (sparse local optima)

Sparse Local Optima - Previous Works



- SPARSELOCALOPT $\Rightarrow$ a kind of fitness landscapes with *sparse deceptive regions* (local optima) and *dense fitness valleys* (Dang et al., 2021).
- *Non-linear non-elitist selection* and *sufficiently high mutation rate* (Dang et al., 2021) $\Rightarrow$
  - Sparse local optimal individuals $\Rightarrow$ higher chance to be selected but only survive a small percentage of such individuals after mutation;
  - Dense fitness valley individuals $\Rightarrow$ less chance of being selected but can have higher chance of surviving mutation.

Sparse Local Optima - Problems

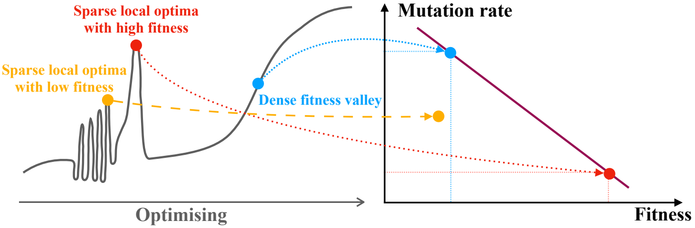


- However,
  - We need know the sparsity of local optima to set the mutation rate;
  - Fitness functions could contain several local optimums with different sparsities.

A New Algorithm: $\mathrm{MOSA\text{-}EA}$

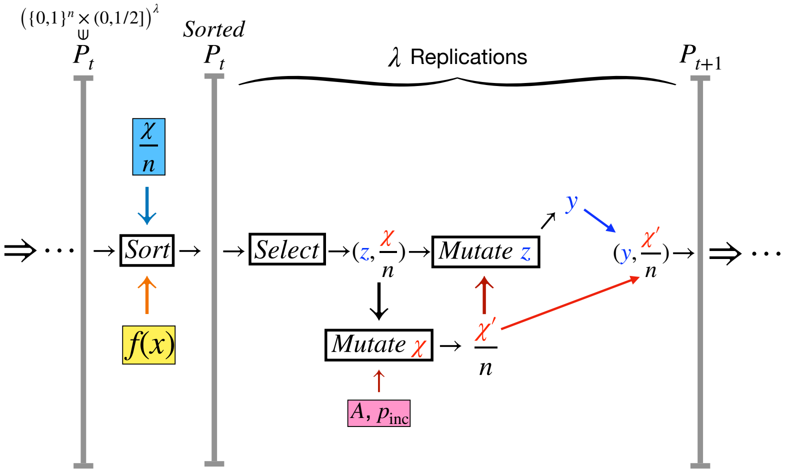
A new EA for single-objective: MOSA-EA

- The *Multi-Objective Self-Adaptive EA* (MOSA-EA)*
  - **Non-elitism**
  - **Self-adaptation**
  - **Multi-objectivisation**



*Implementation can be found in https://github.com/ChengCheng-Qin/mosa-ea.

A Framework of Self-adaptive EAs



Components of MOSA-EA

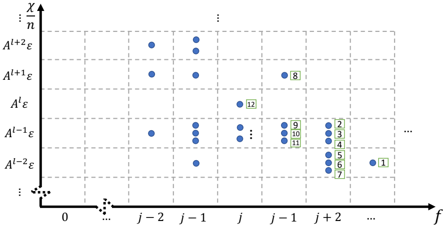


Figure: Fitness-first sorting (Case and Lehre, 2020)

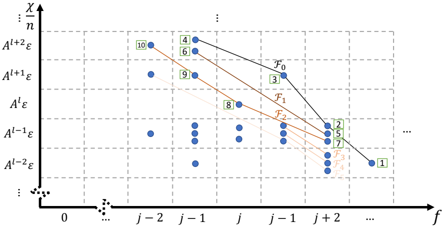


Figure: Multi-objective sorting

- MOSA-EA:
  - Multi-objective sorting mechanism $\Rightarrow$ Strict non-dominated Pareto fronts
  - (μ, λ) selection $\Rightarrow$ from sorted population.
  - Self-adapting mutation rate strategy $\Rightarrow$ New mutation rate χ' is
    - $A\chi$ with probability p_{inc};
    - χ/A otherwise.
  - Bit-wise mutation operator $\Rightarrow$ New population.

Analyses on MOSA-EA

Hyper-parameter Settings in MOSA-EA

- Hyper-parameters p_{inc} and A need not to be tuned carefully.
- We also use the same hyper-parameter setting of the MOSA-EA for all experiments following show that it does not require problem-specific tuning.

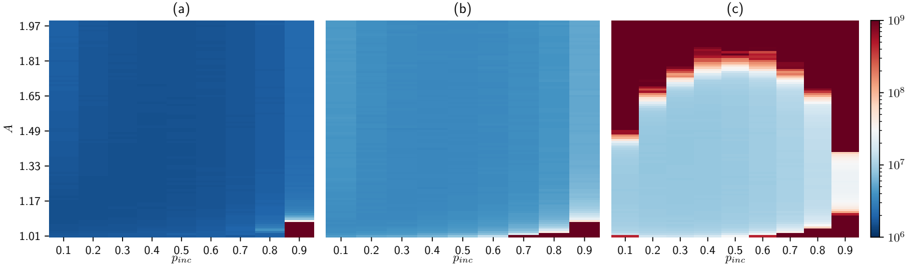


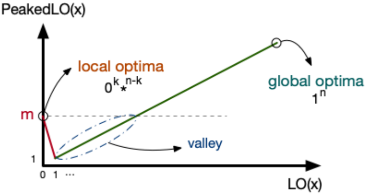
Figure: Median runtimes of the MOSA-EA for different parameters A and p_{inc} on (a) OneMax, (b) LeadingOnes and (c) Funnel over 100 independent runs ($n = 100$).*

*The definition of Funnel can be found in (Dang et al., 2020).

Analyses on MOSA-EA

- Theoretical study: **Escaping local optima** (Runtime analyses)
  - MOSA-EA can efficiently escape an artificial local optimum with unknown sparsity.
  - Other fixed mutation rate EAs fail.
- Empricial study:
  - **Theoretical benchmark functions**
  - **Noisy optimisation**
  - **Complex combinatorial optimisation problems**

Theoretical Study (Lehre and Qin, 2022)



$$\textsc{PeakedLO}_{m,k}(x) = \begin{cases} m & \text{if } x = \{0\}^k *^{n-k} \\ \mathrm{LO}(x) & \text{otherwise.} \end{cases}$$

Table: Runtime analyses of EAs* on $\textsc{PeakedLO}_{m,k}$ (for some constant $c, \delta > 0$)

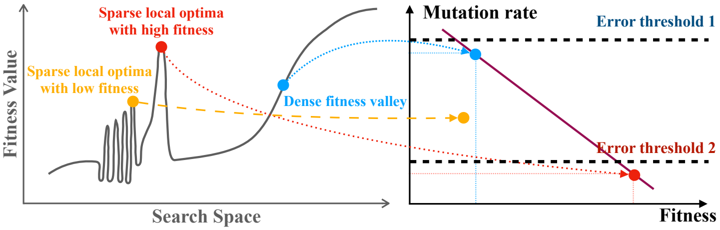
Algorithm	**PeakedLO$_{m,k}$**	**Runtime T**
$(\mu + \lambda)$ EA	Any $k \le n$ and $k, m \in \Omega(n)$	$\Pr(T \le e^{cn}) \le e^{-\Omega(n)}$
(μ, λ) EA	Any $k \le n$ and $k, m \in \Omega(n)$	$\Pr(T \le e^{cn}) \le e^{-\Omega(n)}$ †
2-tour. EA	Any $k < (\ln(3/2) - \delta)\, n$ and $k, m \in \Omega(n)$	$\Pr(T \le e^{cn}) \le e^{-\Omega(\lambda)}$
(μ, λ) MOSA-EA	Any $n \ge k \in \Omega(n)$, $\lceil m \rceil < 2A\,(1 + \ln(p_{\text{inc}})/\ln(\alpha_0) - o(1))\, k$ ‡	$E[T] = O\left(n^2 \log(n)\right)$

*With the initial population $P_0 = \{0^k *^{n-k}\}^\lambda$

† $\lambda, \mu \in \mathrm{poly}(n)$

‡ For some constants $p_{\text{inc}} < 2/5$, $\alpha \ge 4$, $A > 1$ based on restrictions in Theorem 5 of (Lehre and Qin, 2022)

Proof Idea



- The error thresholds for sparse and dense regions are different.
- MOSA-EA maximises fitness and mutation rate on Pareto fronts.
- For each individual, the mutation rate will be closed to its error threshold.
- Individuals with mutation rates larger than error thresholds will "vanish" in the next generation.
- Partition the two-dimensional search space into "fitness levels" and "mutation rate sub-levels"
- Use the level-based theorem (Corus et al., 2018) to derive the runtime.

Empirical Study (Lehre and Qin, 2022)

- MOSA-EA is comparable with or efficient than other heuristic algorithms on theoretical benchmark functions. (Skipping)
- MOSA-EA self-adapts the mutation rate to the noise level in noisy optimisation.
- MOSA-EA outperforms other heuristic algorithms on
  - random NK-Landscape and random k-MaxSat instances.

Noisy Optimisation

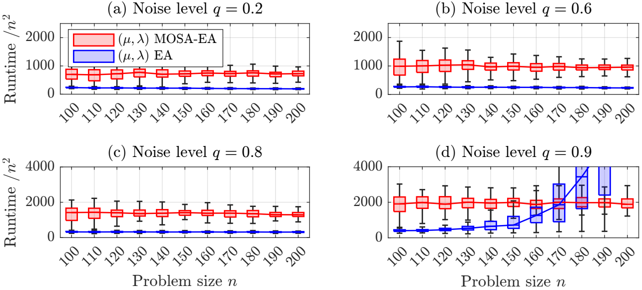


Figure: Runtimes of MOSA-EA and (μ, λ) EA with the fixed mutation rate $\chi/n = 1.386/n$ on LeadingOnes under one-bit noise with noise levels q (Qin and Lehre, 2022).

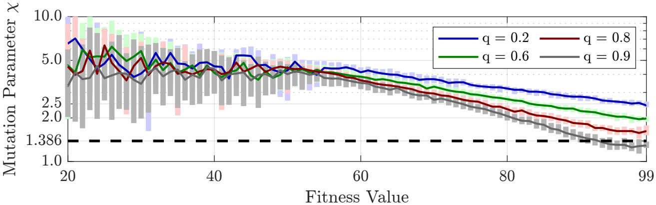


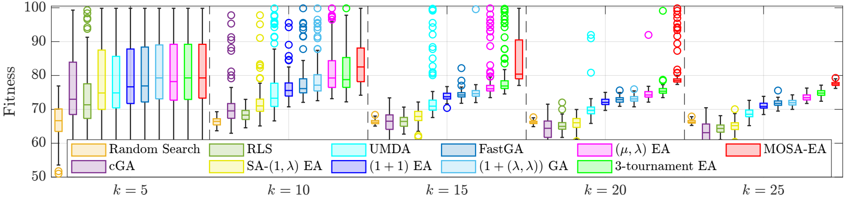
Figure: Real fitness and mutation parameter of the highest real fitness individual per generation of MOSA-EA on LeadingOnes under one-bit noise with noise levels q (Qin and Lehre, 2022).

Complex Combinatorial Optimisation Problems

Contain many local optimums

⇑

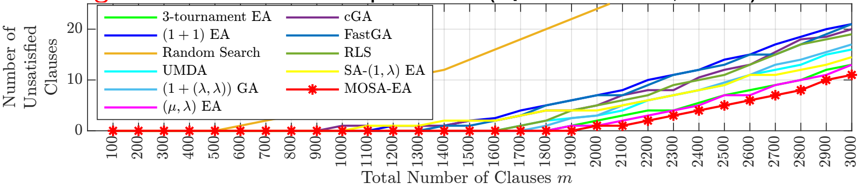
Maximising random NK-LANDSCAPE problems (Qin and Lehre, 2022):



In the complexity class NP

⇑

Minimising random 5-MAXSAT problems (Qin and Lehre, 2022):



Summary

Summary

- **Novelty:**
  - MOSA-EA encodes parameters into individuals.
  - MOSA-EA treats parameter control as another objective.
- **Significance:**
  - MOSA-EA can escape local optima with unknown sparsity
  - MOSA-EA can self-adapt mutation rate to the noise level in noisy optimisation.
  - MOSA-EA can outperform other EAs on complex optimisation problems.
  - MOSA-EA is free to set mutation rate.
- **Next steps:**
  - Performance in more scenarios?
  - Self-adapt more parameters?
  - ...

Thank You

Title: Self-adaptation via Multi-objectivisation:
Multi-Objective Self-Adaptive Evolutionary Algorithms (MOSA-EA)

Contributors: *Xiaoyu Qin* & *Per Kristian Lehre*
{xxq896,p.k.lehre}@cs.bham.ac.uk

Publications:

- Self-adaptation via Multi-objectivisation: An Empirical Study. (with Per Kristian Lehre) To appear in the Parallel Problem Solving from Nature XVII (PPSN '22).
- Self-adaptation via Multi-objectivisation: A Theoretical Study. (with Per Kristian Lehre) In Proceedings of the Genetic and Evolutionary Computation Conference 2022 (GECCO '22).

Code:

References

Case, B. and Lehre, P. K. (2020). Self-adaptation in non-Elitist Evolutionary Algorithms on Discrete Problems with Unknown Structure. *IEEE Transactions on Evolutionary Computation*, pages 1–1.

Corus, D., Dang, D.-C., Eremeev, A. V., and Lehre, P. K. (2018). Level-Based Analysis of Genetic Algorithms and Other Search Processes. *IEEE Transactions on Evolutionary Computation*, 22(5):707–719.

Dang, D.-C., Eremeev, A., and Lehre, P. K. (2020). Escaping Local Optima with Non-Elitist Evolutionary Algorithms. In *Proceedings of AAAI 2021*. AAAI Press, Palo Alto, California USA.

Dang, D.-C., Eremeev, A., and Lehre, P. K. (2021). Non-elitist Evolutionary Algorithms Excel in Fitness Landscapes with Sparse Deceptive Regions and Dense Valleys. In *Proceedings of the Genetic and Evolutionary Computation Conference*, Lille, France. ACM.

Doerr, B. (2022). Does Comma Selection Help to Cope with Local Optima? *Algorithmica*.

Doerr, B. and Doerr, C. (2020). Theory of parameter control for discrete black-box optimization: Provable performance gains through dynamic parameter choices. *Theory of Evolutionary Computation*, pages 271–321. Publisher: Springer.

Eiben, A., Hinterding, R., and Michalewicz, Z. (1999). Parameter control in evolutionary algorithms. *IEEE Transactions on Evolutionary Computation*, 3(2):124–141.

Jagerskupper, J. and Storch, T. (2007). When the Plus Strategy Outperforms the Comma Strategyand When Not. In *2007 IEEE Symposium on Foundations of Computational Intelligence*, pages 25–32, Honolulu, HI. IEEE.

Lehre, P. K. and Qin, X. (2022). Self-adaptation via Multi-objectivisation: A Theoretical Study. In *Proceedings of the Genetic and Evolutionary Computation Conference*, Boston USA. ACM.

Qin, X. and Lehre, P. K. (2022). Self-adaptation via Multi-objectivisation: An Empirical Study. In *Parallel Problem Solving from Nature – PPSN XVII*, Cham. Springer International Publishing.