Self-adaptation via Multi-objectivisation: A Theoretical Study

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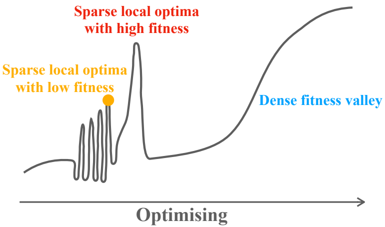
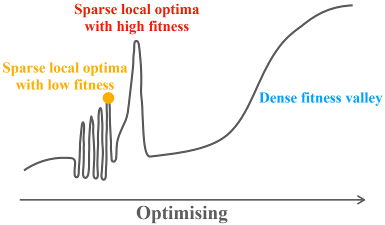
Motivation



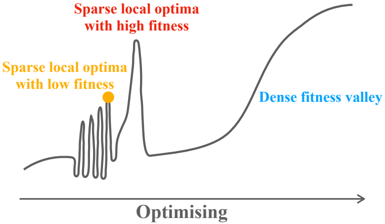
- Elitist EAs can get stuck on local optima (Jagerskupper and Storch, 2007; Dang et al., 2020; Doerr, 2022; Dang et al., 2021).
- *Sparse deceptive regions* (sparse local optima)

Motivation - Previous Works



- SparseLocalOpt $\Rightarrow$ a kind of fitness landscapes with *sparse deceptive regions* (local optima) and *dense fitness valleys* (Dang et al., 2021).
- *Non-linear non-elitist selection* and *sufficiently high mutation rate* (Dang et al., 2021) $\Rightarrow$
  - Sparse local optimal individuals $\Rightarrow$ higher chance to be selected but only survive a small percentage of such individuals after mutation;
  - Dense fitness valley individuals $\Rightarrow$ less chance of being selected but can have higher chance of surviving mutation.

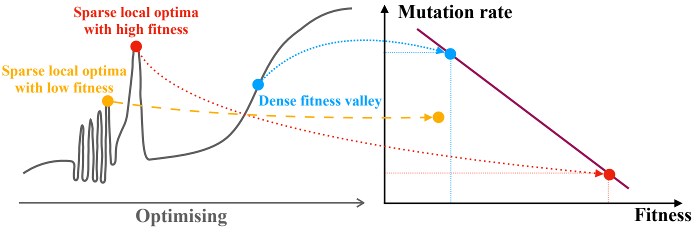
Motivation - Problems



- However,
  - We need know the sparsity of local optima to set the mutation rate;
  - Fitness functions could contain several local optimums with different sparsities.

Contribution: A new EA for single-objective (MOSA-EA)

- The *Multi-Objective Self-Adaptive EA* (MOSA-EA)[3]
  - **Non-elitism**
  - **Self-adaptation**
  - **Multi-objectivisation**



[3] Implementation can be found in https://github.com/ChengCheng-Qin/mosa-ea.

Contribution: Analyses on MOSA-EA

- Theoretical study: **Escaping local optima** (Runtime analyses)
  - MOSA-EA can efficiently escape an artificial local optimum with unknown sparsity.
  - Other fixed mutation rate EAs fail.
- Empricial study: **Complex combinatorial optimisation problems**
  - MOSA-EA can outperforms a range of EAs on random NK-Landscape and k-Sat instances.

MOSA-EA

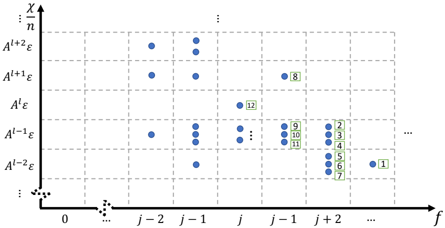


Figure: Fitness-first sorting (Case and Lehre, 2020)

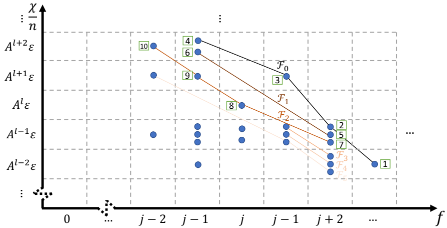
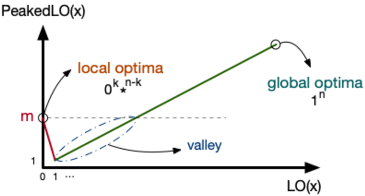


Figure: Multi-objective sorting

- **MOSA-EA:**
  - Multi-objective sorting mechanism $\Rightarrow$ Strict non-dominated Pareto fronts
  - (μ, λ) selection $\Rightarrow$ from sorted population.
  - Self-adapting mutation rate strategy $\Rightarrow$ New mutation rate χ' is
    - $A\chi$ with probability p_{inc};
    - χ/A otherwise.

Main Results



$$\textsc{PeakedLO}_{m,k}(x) = \begin{cases} m & \text{if } x = \{0\}^k *^{n-k} \\ \mathrm{LO}(x) & \text{otherwise.} \end{cases}$$

Table: Runtime analyses of EAs[1] on $\textsc{PeakedLO}_{m,k}$ (for some constant $c, \delta > 0$)

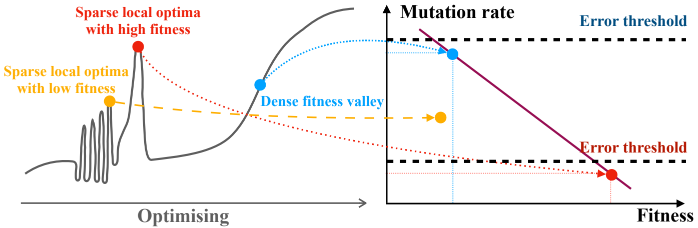
Algorithm	PeakedLO$_{m,k}$	Runtime T
$(\mu + \lambda)$ EA	Any $k \leq n$ and $k, m \in \Omega(n)$	$\Pr\left(T \leq e^{cn}\right) \leq e^{-\Omega(n)}$
(μ, λ) EA	Any $k \leq n$ and $k, m \in \Omega(n)$	$\Pr\left(T \leq e^{cn}\right) \leq e^{-\Omega(n)}$ [2]
2-tour. EA	Any $k < (\ln(3/2) - \delta)\, n$ and $k, m \in \Omega(n)$	$\Pr\left(T \leq e^{cn}\right) \leq e^{-\Omega(\lambda)}$
(μ, λ) MOSA-EA	Any $n \geq k \in \Omega(n)$, $\lceil m \rceil < 2A\left(1 + \ln(p_{\mathrm{inc}})/\ln(\alpha_0) - o(1)\right) k$ [3]	$E[T] = O\left(n^2 \log(n)\right)$

[1]With the initial population $P_0 = \{0^k *^{n-k}\}^\lambda$

[2]$\lambda, \mu \in \mathrm{poly}(n)$

[3]For some constants $p_{\mathrm{inc}} < 2/5$, $\alpha \geq 4$, $A > 1$ based on restrictions in Theorem 5 of the Paper.

Proof Idea



- The error thresholds for sparse and dense regions are different.
- MOSA-EA maximises fitness and mutation rate on Pareto fronts.
- For each individual, the mutation rate will be closed to its error threshold.
- Individuals with mutation rates larger than error thresholds will "vanish" in the next generation.
- Partition the two-dimensional search space into "fitness levels" and "mutation rate sub-levels"
- Use the level-based theorem (Corus et al., 2018) to derive the runtime.

Supplemental Experimental Results

- MOSA-EA outperforms other heuristic algorithms on
  - random NK-LANDSCAPE (maximisation, above) and
  - random k-SAT (minimisation, below).
- More empirical analyses will be published in (Qin & Lehre, PPSN'22).

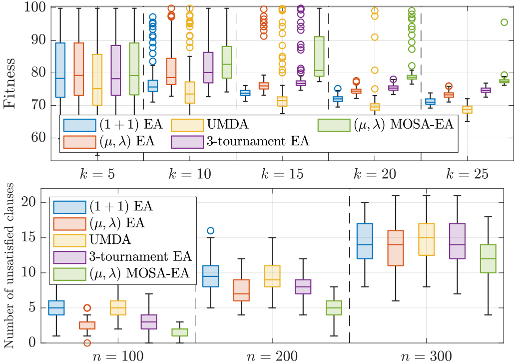


Figure: The best fitness value achieved in 10^8 fitness evaluations on 100 random NK-LANDSCAPE ($n = 100$) (above) and k-SAT ($k = 5$) (below) instances

Summary

- **Novelty:**
  - MOSA-EA treats parameter control as another objective.
- **Significance:**
  - MOSA-EA can escape local optima with unknown sparsity
  - MOSA-EA can outperform other EAs on complex optimisation problems.
  - MOSA-EA is free to set mutation rate.
- **Next steps:**
  - Performance in more scenarios?
  - Self-adapt more parameters?
  - ...

Thank you

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Code can be found here:

References

Case, B. and Lehre, P. K. (2020). Self-adaptation in non-Elitist Evolutionary Algorithms on Discrete Problems with Unknown Structure. *IEEE Transactions on Evolutionary Computation*, pages 1–1.

Corus, D., Dang, D.-C., Eremeev, A. V., and Lehre, P. K. (2018). Level-Based Analysis of Genetic Algorithms and Other Search Processes. *IEEE Transactions on Evolutionary Computation*, 22(5):707–719.

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Dang, D.-C., Eremeev, A., and Lehre, P. K. (2021). Non-elitist Evolutionary Algorithms Excel in Fitness Landscapes with Sparse Deceptive Regions and Dense Valleys. In *Proceedings of the Genetic and Evolutionary Computation Conference*, Lille, France. ACM.

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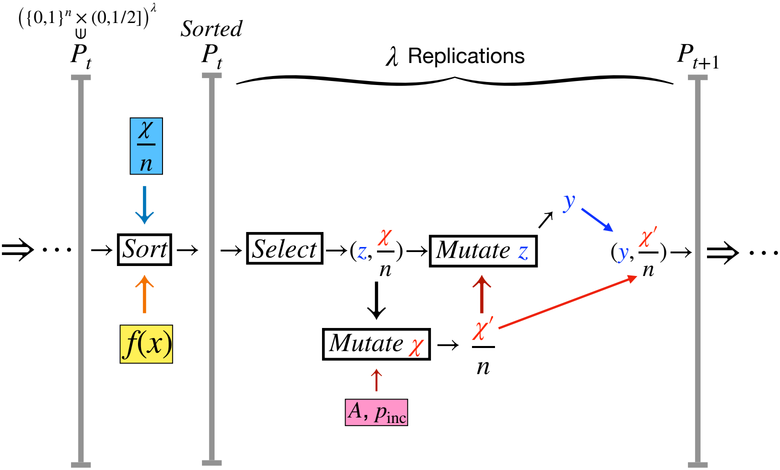
A Framework of Self-adaptive EAs



More Experimental Results in (Qin & Lehre, PPSN '22)

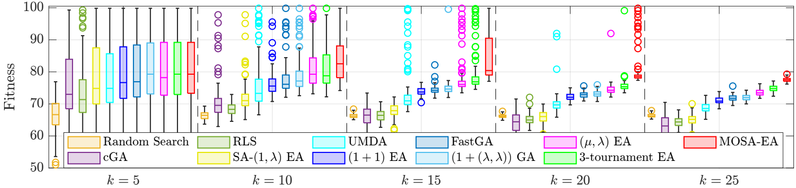


Figure: The highest fitness values found in the end of runs in 10^8 fitness evaluations on 100 random NK-LANDSCAPE instances with different k ($n = 100$).

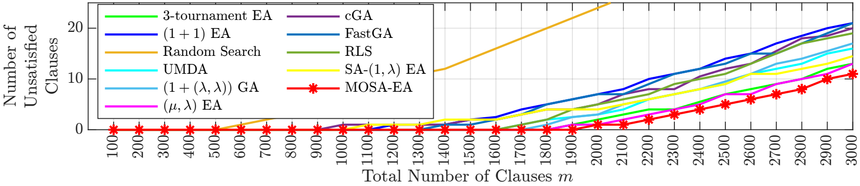


Figure: The medians of the smallest number of unsatisfied clauses found in 10^8 fitness evaluations on 100 random k-SAT instances with different total numbers of clauses m ($k = 5$, $n = 100$).